

# Comparison of 2 various approaches to determine subpixel coordinates of corner point in static scene

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*Determination of corner point position in subpixel accuracy has a high significance in many practical applications. This paper deals with subpixel accuracy of Harris corner detector and comparison of 2 approaches to specify the results. For our experiment we used very accurate pointing device and there was set of images for every position taken. The main goal is to use several images in static scene for getting more accurate and more robust results. First approach is based on averaging of found corner point coordinates among all images in one set. The second one is about averaging of brightness intensity for all images in one set before we apply the detector. We compared both these approaches and there were appropriate statistical analysis performed. All the results are illustrated in graphs and listed in tables. This comparison and our study could be convenient in many types of applications and measurements in case the precision is a key.*

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## Introduction

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The area of corner point detection is well known and very often used in many practical tasks. We can imagine corner point as a point, where at least two edges are intersected, point around which is high change of brightness intensity in all directions or point having the smallest radius of curvature for example. Many corner detectors were invented over the years and Harris corner detector is one the most famous. But there are situations when pixel accuracy is not sufficient enough. For that reason we can use some mathematical techniques and to find chosen features in subpixel accuracy. This paper deals with subpixel detection and possibility to improve detection accuracy by using multiple images of the same static scene. We tested 2 approaches how to statistically improve precision of subpixel detection and compared the results we obtained. All the results are presented in form of graphs or tables of course. This study could be helpful in case we need very precise results and gives us the answer what the limits of camera measurements in real conditions are. The reference [1] deals with precision of subpixel detection for example.

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## Harris corner detector

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The principle of Harris corner detector was first time mentioned in [2]. The main idea is to find the minimum of brightness intensity difference between chosen parts of an image (marked as  $W$ ) and shifted parts of image in all directions. There is first-order Taylor series approximation for that purpose used. First step is determination of matrix  $M$  as it is shown in 1.

$$M = \sum_W \left\{ \begin{bmatrix} I_x \\ I_y \end{bmatrix} * [I_x, I_y] \right\} = \begin{bmatrix} A & C \\ C & B \end{bmatrix} \quad (1)$$

The variable  $I_x$  means approximation of derivation (also known as difference) in horizontal direction. The sign  $I_y$  has the same meaning in vertical direction. There are suitable convolution masks for computing these differences used. The window  $W$  has usually size 3x3, 5x5 or 7x7 pixels. The matrix  $M$  is calculated for every pixel in image of course

Next step is definition of values contained in matrix  $C$ . This matrix has the same size as tested image. There were many publications and mathematical equations for calculation of  $C$  values formulated and we decided to choose the one mentioned in [3] as it is shown in formula 2.

$$C(x, y) = \min(\lambda_1, \lambda_2) \quad (2)$$

The variables  $x$  and  $y$  are coordinates of particular pixel in corresponding directions. The signs  $\lambda_1$  and  $\lambda_2$  are eigenvalues of 2x2 sized matrixes  $M$  and they are computed by using equation

3.

$$\lambda_{1,2} = \frac{1}{2} \left[ (A + B) \pm \sqrt{4C^2 + (A - B)^2} \right] \quad (3)$$

The elements of equations  $A$ ,  $B$ ,  $C$  and  $D$  are determined from matrix  $M$ .

Last step is looking for the elements in matrix  $M$  having the highest values. These points are marked as corner points. It is necessary to use global and local thresholding of course.

As it is obvious, this algorithm can be used to find corner points in pixel accuracy. If we consider subpixel detection, the first step is usually to find pixel coordinates. Next step is to choose suitable window (area) surrounding found corner point and then we can use specific subpixel detector or algorithm to obtain the more precise subpixel coordinates inside this area.

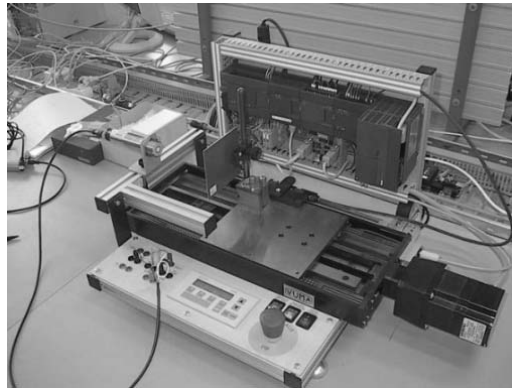
There is a possibility to use the principle of Harris detector in subpixel detection if we interpolate the brightness intensity between the pixels. For that reason bilinear or bicubic interpolation is usually used. We may use step 0.01 or even 0.001 pixels between pixels and state the brightness intensity values for all these shifts. It means that 1 pixel contains 100 or 1000 subpixels in this case. So we can reach 100 or 1000 times higher accuracy as initially. The rest of the procedure is exactly the same as we described before and we can use the Harris detector usual way.

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### Experimental tests

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The tests we have taken are very similar to the tests described in [4]. We have chosen very accurate (up to 0.01 mm) pointing device as it is illustrated in figure 1. We fixed a small picture of chessboard segment (because it contains easy detected  $X$ -corner points) to pointing device and we set small shifts on device (the orders of hundredths of millimetre). Then we took series of images with high-resolution camera and we found subpixel coordinates ( $x$  axis is sufficient because of movement in horizontal direction) from every single image from series. There were 30 images for 10 different positions taken in our measurement. Our research is based on comparison



**Figure 1.** The chosen pointing device.

of 2 approaches. In first case we apply Harris detector on every image from set. Then we get the final coordinates by using averaging of results we got. The second approach deals with averaging of brightness intensity for all images in one set before we apply the detector. There were other statistical analysis performed and all the results are shown in graphs and tables. All shifts on pointing device are recalculated in pixels, not hundredths of millimetres as originally. It is important to notice, that we got the images in standard lightning conditions. For that reason, there is an influence of brightness intensity changes.

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### Experimental results

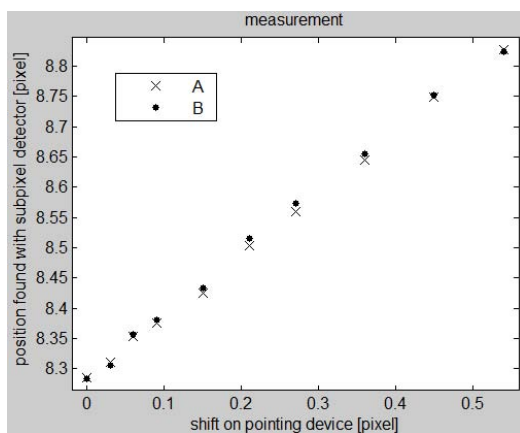
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There are two types of values in figure 2 and they are marked as  $A$  and  $B$  in legend. The sign  $A$  is arithmetic mean of all  $x$ -coordinates we obtained for one position (one shift) of pointing

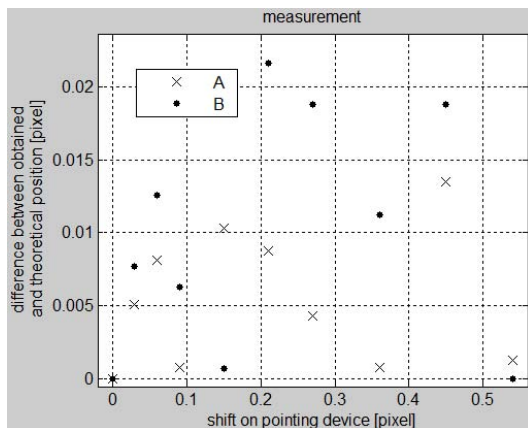
device by using first type of approach. These coordinates mean absolute position of detected corner point in chosen window. The  $B$  has the same meaning consider second approach we mentioned. This notation is used in whole paper. It is important to notice, that we have not used perfectly calibrated camera and we have not known the exact focal length of camera. For that reason we modified the last obtained coordinate to its corresponding theoretical coordinate and we multiplied found coordinates for every position by these correction coefficients. As it is possible to see, there is no significant difference between both approaches. The exact values of our results are presented in table 1 and table 2.

Figure 3 deals with difference between obtained (stated arithmetic mean) and theoretical position of corner point. It is obvious that maximal difference in case of approach  $A$  is less than 15 thousandth of pixel. There is maximal difference a little bit more than 20 thousandths of pixel consider approach  $B$ .

The all data we obtained are listed in table 1 and table 2. The signs  $A$  and  $B$  are representing the approaches we mentioned before. The label marked as regular means we made our analysis from wholes sets of data. Likewise note truncated says we used truncated data in our tests.



**Figure 2.** Horizontal coordinates for every position obtained by method  $A$  and  $B$ .



**Figure 3.** The difference between obtained and theoretical coordinates.

It means that some of extreme elements from sets of coordinates or brightness intensities values were neglected. As you can see there are standard deviations  $\sigma$  and  $x$ -coordinates for every position written in this table.

If we consider approach  $A$ , the average value of  $\sigma$  is about 18 thousandths of pixel in regular version and about 6 thousandths in truncated version. There are the averages values of  $\sigma$  approximately 3 times lower for both versions of data in approach  $B$  as it was in case of approach  $A$ . It is interesting that using of truncated data can reduce the values of  $\sigma$  to one-third. The usage of approach  $B$  and truncated data seem to be the best option according the  $\sigma$  comparison.

We performed our experiments in standard lightning conditions. Brightness intensity changing is the reason why there are differences between data we observed.

**Table 1.** The result for regular and truncated values in approach *A*

	<i>A</i>		<i>A</i>	
	<i>regular</i>	<i>regular</i>	<i>truncated</i>	<i>truncated</i>
<i>shift[pixel]</i>	<i>x[pixel]</i>	<i>σ[pixel]</i>	<i>x[pixel]</i>	<i>σ[pixel]</i>
0	8.2853	0.0200	8.2850	0.0053
0.03	8.3102	0.0174	8.3115	0.0070
0.06	8.3534	0.0187	8.3556	0.0032
0.09	8.3761	0.0180	8.3746	0.0084
0.15	8.4250	0.0221	8.4314	0.0099
0.21	8.5041	0.0180	8.5020	0.0048
0.27	8.5596	0.0208	8.5575	0.0057
0.36	8.6445	0.0144	8.6433	0.0032
0.45	8.7488	0.0105	8.7480	0.0042
0.54	8.8266	0.0159	8.8250	0.0048
		<b>0.0176</b>		<b>0.0057</b>

**Table 2.** The result for regular and truncated values in approach *B*

	<i>B</i>		<i>B</i>	
	<i>regular</i>	<i>regular</i>	<i>truncated</i>	<i>truncated</i>
<i>shift[pixel]</i>	<i>x[pixel]</i>	<i>σ[pixel]</i>	<i>x[pixel]</i>	<i>σ[pixel]</i>
0	8.2840	0.0076	8.2820	0.0014
0.03	8.3063	0.0089	8.3070	0.0016
0.06	8.3556	0.0039	8.3570	0.0019
0.09	8.3803	0.0053	8.3806	0.0018
0.15	8.4333	0.0042	8.4333	0.0012
0.21	8.5156	0.0041	8.5152	0.0013
0.27	8.5728	0.0042	8.5735	0.0011
0.36	8.6552	0.0030	8.6540	0.0011
0.45	8.7528	0.0043	8.7512	0.0008
0.54	8.8240	0.0051	8.8220	0.0013
		<b>0.0051</b>		<b>0.0014</b>

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## Conclusion

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This paper has dealt with sensitivity of subpixel detection and comparison of 2 approaches to determine subpixel coordinates of corner point in static scene. We have decided to test Harris corner detector because of its very good detection qualities and possibility to be applied in real conditions. We implemented our own experiments where we chose very accurate pointing device. Then we were changing positions of chessboard fragment image and we were observing how our obtained coordinates are changing. We performed statistic analysis and there were corresponding graphs and tables created. For these tasks there were appropriate computational scripts in development environment Matlab programmed.

As it is shown in figure 2 the absolute positions found using approach *A* and *B* are very similar. It is obvious that there is no significant difference between them.

Figure 3 deals with difference between theoretical and obtained (arithmetical mean) position of corner point. Maximal difference in case of approach *A* is slightly less than in case of *B*.

The all data are listed in table 1 and table 2. If we consider the values of  $\sigma$  we can see that approach B has 3 times better results than approach A. It could be convenient to use truncated data in these kinds of tests, because of 3 times lower values of  $\sigma$  compared to regular set of data. The difference between both approaches is small enough and there is possibility to use both of them in very accurate measurements. But advantage of approach B is its much lower computational complexity.

There are a few limitations in our approach. Firstly, the camera has limited resolution and bit depth. The pointing device is not absolutely precise and we are not able to take infinity number of images. Because of that our statistic analyses is not as accurate as could be. Take multiple images is reasonable only for static scene. However, in case of dynamic scene there are some other issues we need to consider.

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